Incomplete Information and Investment Inaction

Kansas City Fed Research Dept. Brown Bag: August 27, 2025

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*The views in this paper are solely the authors' responsibility and should not reflect the views of the Federal Reserve Bank of Kansas City or the Board of Governors of the Federal Reserve System.

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- Both frictions are important, but studied individually. Do they interact?

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- Several lines of active research trying to resolve this tension, e.g. production networks (Winberry and vom Lehn 2025)

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- Firms with worse information behave as predicted by model

Theory

Firms' Problem

- Atomistic firms face simple investment problem
- Produce using capital K and stochastic productivity A by

$$F(A,K)=A^{1-\alpha}K^{\alpha}$$

• Log productivity *a* follows a random walk:

$$da = \sigma_a dW^a$$

• Investment *I* is irreversible. Conditional on investing, profits are

$$\pi = A^{1-\alpha}K^{\alpha} - \psi I$$

The law of motion for capital is

$$dK = I - \delta K dt$$

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- Effect of incomplete information? It determines the inaction region

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- â follows a random walk with the same properties as a

▶ Nowcast Behavior

• We work with normalized capital $x \equiv k - a$ as in Stokey (2008) \Longrightarrow renormalize value function as $V(e^x)$

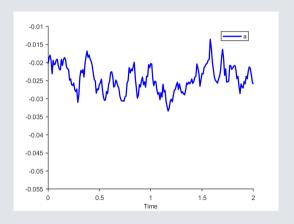
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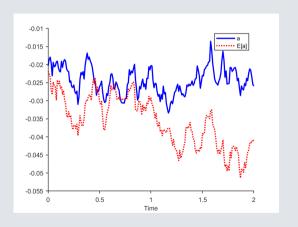
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- ullet Firm maximizes expected value function $\hat{V}(e^{\hat{x}}) = \mathbb{E}[V(e^{x})|e^{\hat{x}}]$
- We show that the optimum is characterized by usual value-matching and super contact conditions, except applied to \hat{V} :

$$\hat{V}'(e^{\hat{b}}) = \psi$$
 $\lim_{e^{\hat{x}} \to \infty} \hat{V}'(e^{\hat{x}}) = 0$ $\hat{V}''(e^{\hat{b}}) = 0$ $\lim_{e^{\hat{x}} \to \infty} \hat{V}''(e^{\hat{x}}) = 0$

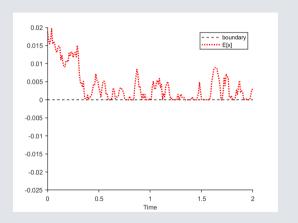




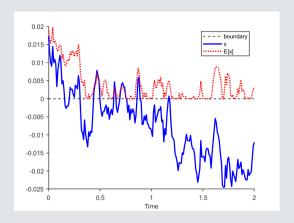
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- Actual norm. capital x follows $x = k a = \hat{x} + \hat{a} a$

Micro-Level Implications 1: Reduced Inaction

1. Information friction increases the incentive to invest

$$\hat{b} = \underbrace{b^{FI}}_{\text{Full Info.}} + \frac{\alpha^2}{2(1-\alpha)} \underbrace{\frac{\tau \sigma_{\text{a}}^2 \sigma_{\text{n}}^2}{\sigma_{\text{a}}^2 + \sigma_{\text{n}}^2}}_{\text{Var}[u]}$$

- Greater noise $(\sigma_n \uparrow)$ or delay $(\tau \uparrow)$ raise boundary \hat{b}
- Contrasts with traditional uncertainty channel: $\sigma_a \uparrow \Longrightarrow b^{FI} \downarrow$
- Why? An Oi-Hartman-Abel effect:
 - ullet MPK is convex in log productivity. Firms: risk-loving on normalized capital x
 - ullet Friction acts as a mean preserving spread on x

Micro-Level Implications 2: Attenuated Shocks

2. Information friction reduces elasticity of forecasts to productivity shocks

$$rac{d}{d a_{t-h}} \mathbb{E}[a_t | \Omega_t] = egin{cases} \gamma & 0 \leq h < au \ 1 & h \geq au \end{cases}$$

where

$$\gamma \equiv \frac{\sigma_a^2}{\sigma_a^2 + \sigma_n^2} < 1$$

Testable predictions for later: worse information associated with

- Lower inaction rate, conditional on firm size
- Lower sensitivity of investment to productivity shocks

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• ODE solution:

$$h(\hat{x}) = \rho e^{-\rho(\hat{x}-\hat{b})}, \quad \text{where} \quad \rho \equiv \frac{\delta}{\sigma_a^2} + \sqrt{\frac{\delta^2}{\sigma_a^4} + 2\frac{\eta}{\sigma_a^2}}$$

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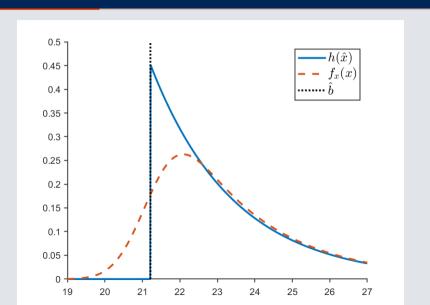
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• Actual $x = \hat{x} + u$ is more dispersed

Stationary Distributions: Expected & Actual Normalized Capital



• Joint distribution $f_{k,\hat{x}}(k,\hat{x})$ satisfies (+ boundary conditions)

$$0 = \frac{\sigma_a^2}{2} \partial_{\hat{x}}^2 f_{k,\hat{x}} + \delta \left(\partial_{\hat{x}} f_{k,\hat{x}} + \partial_k f_{k,\hat{x}} \right) - \eta f_{k,\hat{x}}$$

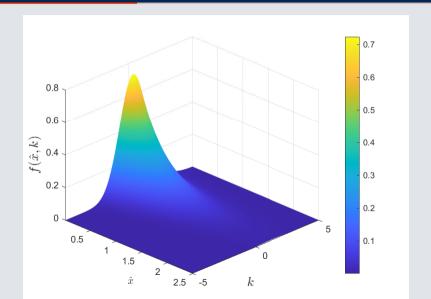
• PDE solution:

$$f_{k,\hat{x}}(k,\hat{x}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{-\mathcal{N}(\xi)}{\frac{\sigma_a^2}{2} \lambda_-(\xi) + \delta} e^{i\xi k + \lambda_-(\xi)(\hat{x} - \hat{b})} d\xi$$

where $\mathcal{N}(\xi)$ denotes the Fourier transform of $\eta\phi\left(\frac{k}{\varsigma}\right)$ and

$$\lambda_-(\xi) \equiv rac{-\delta - \sqrt{\delta^2 + 2\sigma_a^2(i\delta\xi + \eta)}}{\sigma_a^2}$$

Stationary Distribution: Capital & Expected Norm. Capital



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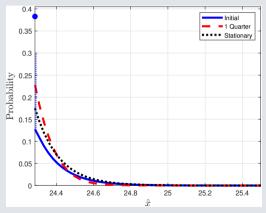
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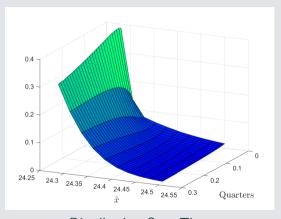
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- 4. Information friction attenuates aggregate responses to productivity shocks:

$$\widehat{IRF}_k(t) = \gamma \widehat{IRF}_k^{FI}(t)$$
 $\gamma = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_n^2}$

Aggregate Response of $\hat{x} = k - \hat{a}$ to a Productivity Shock

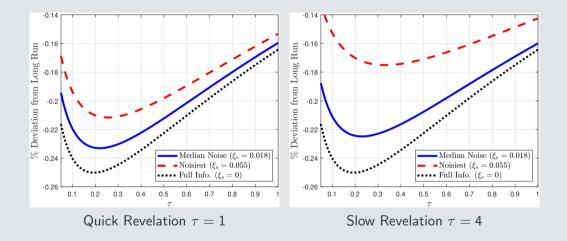


Distribution After Shock



Distribution Over Time

Info. Friction Attenuates Aggregate Response to Shocks



Validation with Firm-level Data

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 - Merged dataset contains firms with at least 1 billion JPY in registered capital

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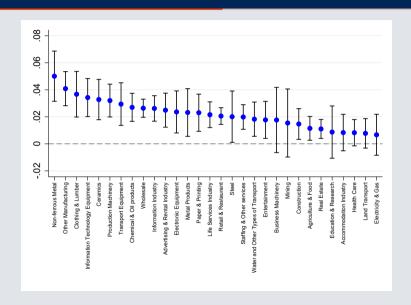
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- Positive $\xi_s \implies$ forecast underreaction

Attenuation Coefficients across Industries



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- γ_t : time fixed effects

- Do we observe more investment inaction for firms in industries with more severe information frictions?
- We estimate

inaction_{it} =
$$\alpha \xi_s + \Gamma z_{it} + \Lambda \zeta_s + \gamma_t + \epsilon_{it}$$

- z_{it}: firm-level controls
- ζ_s : industry-level controls
- γ_t : time fixed effects
- Standardize ξ_s

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- z_{it}: firm-level controls
- ζ_s : industry-level controls
- γ_t : time fixed effects
- Standardize ξ_s
- α is the coefficient of interest
- We calibrate & simulate our model (& match ξ_s distrib.) for comparison.



				inacti	on=1			
			Da	nta			Мс	del
ξς	-0.076**	-0.079***	-0.054**	-0.069**	-0.039*	-0.051**	-0.013	-0.011
	(0.028)	(0.026)	(0.025)	(0.026)	(0.020)	(0.021)	()	()
$a_{i,t}$	0.039	0.059*	0.104***	0.113***	0.091**	0.099***	-0.206	-0.298
	(0.034)	(0.031)	(0.038)	(0.033)	(0.033)	(0.032)	()	()
$k_{i,t-1}$		-0.050***	-0.049***	-0.044***	-0.041***	-0.039***		-0.458
		(0.009)	(0.009)	(0.007)	(800.0)	(0.007)		()
$m_{i,t}$			-0.026	-0.045***	-0.015	-0.030**		
			(0.021)	(0.016)	(0.019)	(0.014)		
cap share $_s$				-0.549*		-0.366		
				(0.314)		(0.304)		
growth vol_s					1.016***	0.870***		
					(0.279)	(0.278)		
Time FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	99027	99027	86294	86294	86294	86294	14291997	14291997
adj. R ²	0.038	0.069	0.063	0.089	0.078	0.095	0.116	0.180

				inacti	on = 1			
			Da	nta			Мс	del
ξ_s	-0.076**	-0.079***	-0.054**	-0.069**	-0.039*	-0.051**	-0.013	-0.011
	(0.028)	(0.026)	(0.025)	(0.026)	(0.020)	(0.021)	()	()
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	(0.034)	(0.031)	(0.038)	(0.033)	(0.033)	(0.032)	()	()
$k_{i,t-1}$		-0.050***	-0.049***	-0.044***	-0.041***	-0.039***		-0.458
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			Da	nta			Mo	Model			
ξ_s	-0.076**	-0.079***	-0.054**	-0.069**	-0.039*	-0.051**	-0.013	-0.011			
	(0.028)	(0.026)	(0.025)	(0.026)	(0.020)	(0.021)	()	()			
$a_{i,t}$	0.039	0.059*	0.104***	0.113***	0.091**	0.099***	-0.206	-0.298			
	(0.034)	(0.031)	(0.038)	(0.033)	(0.033)	(0.032)	()	()			
$k_{i,t-1}$		-0.050***	-0.049***	-0.044***	-0.041***	-0.039***		-0.458			
		(0.009)	(0.009)	(0.007)	(800.0)	(0.007)		()			
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1 SD in $\xi_s \Rightarrow 5.1$ p.p. (14%) less inaction

- Do we see lower investment sensitivity to productivity shocks in industries with stronger information frictions?
- We estimate

inaction_{it} =
$$\beta(w_{it} \times \xi_s) + \gamma w_{it} + \Gamma z_{it} + \gamma_i + \gamma_{st} + \epsilon_{it}$$

- w_{it} : productivity shock (random walk or AR(1))
- z_{it}: firm-level controls
- γ_i : firm fixed effects
- γ_{st} : industry-time fixed effects
- Standardize ξ_s

		on=1				
]	Model			
$\xi_s \times w_{i,t}$	0.010**	0.011**	0.011**	0.010**	0.012	0.013
	(0.005)	(0.005)	(0.005)	(0.005)	()	()
Wit	-0.036	-0.030	-0.036	-0.029	-0.188	-0.188
	(0.031)	(0.031)	(0.032)	(0.032)	()	()
a_{it-1}	-0.028**	-0.015	-0.029**	-0.016	-0.670	-0.670
	(0.012)	(0.012)	(0.011)	(0.011)	()	()
Productivity	Rand. Walk	Rand. Walk	Auto. Reg(1)	Auto. Reg(1)	Rand. Walk	Rand. Walk
Firm FE	Y	Υ	Υ	Υ	Υ	Υ
Time FE	Y	Υ	Υ	Υ	Υ	Υ
Industry-Time FE	N	Υ	N	Υ	N	Υ
N	84656	84656	84313	84313	14274640	14274640
adj. R^2	0.446	0.451	0.446	0.451	0.450	0.450

			inactio	on = 1		
]	Model			
$\xi_s \times w_{i,t}$	0.010**	0.011**	0.011**	0.010**	0.012	0.013
	(0.005)	(0.005)	(0.005)	(0.005)	()	()
Wit	-0.036	-0.030	-0.036	-0.029	-0.188	-0.188
	(0.031)	(0.031)	(0.032)	(0.032)	()	()
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	(0.012)	(0.012)	(0.011)	(0.011)	()	()
Productivity	Rand. Walk	Rand. Walk	Auto. Reg(1)	Auto. Reg(1)	Rand. Walk	Rand. Walk
Firm FE	Y	Υ	Υ	Υ	Υ	Υ
Time FE	Y	Υ	Υ	Υ	Υ	Υ
Industry-Time FE	N	Υ	N	Υ	N	Υ
N	84656	84656	84313	84313	14274640	14274640
adj. R^2	0.446	0.451	0.446	0.451	0.450	0.450

Dampened inaction responses to prod. shocks in industries with higher ξ

			inactio	on=1		
]	Model			
$\xi_s \times w_{i,t}$	0.010**	0.011**	0.011**	0.010**	0.012	0.013
	(0.005)	(0.005)	(0.005)	(0.005)	()	()
Wit	-0.036	-0.030	-0.036	-0.029	-0.188	-0.188
	(0.031)	(0.031)	(0.032)	(0.032)	()	()
a_{it-1}	-0.028**	-0.015	-0.029**	-0.016	-0.670	-0.670
	(0.012)	(0.012)	(0.011)	(0.011)	()	()
Productivity	Rand. Walk	Rand. Walk	Auto. Reg(1)	Auto. Reg(1)	Rand. Walk	Rand. Walk
Firm FE	Y	Υ	Υ	Υ	Υ	Υ
Time FE	Y	Υ	Υ	Υ	Υ	Υ
Industry-Time FE	N	Υ	N	Υ	N	Υ
N	84656	84656	84313	84313	14274640	14274640
adj. R^2	0.446	0.451	0.446	0.451	0.450	0.450

1 SD in $\xi_s \Rightarrow$ reduces prod shock response by $\sim 1/3$

Conclusions

- Information and investment frictions interact in rich ways
- Parsimonious model delivers testable predictions, consistent with the data
- Information frictions are easily incorporated into continuous time inaction models (there are many applications beyond investment)
- An alternative structure for investment frictions:
 - Old paradigm: fixed costs to get inaction, + large or convex adjustment costs to get attenuation
 - New paradigm: irreversibility to get inaction, + information frictions to get attenuation
- Strong empirical evidence, and robust to many alternative specifications

Appendix

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How Do Firms Nowcast?

Lemma (1.a)

For a firm with information set $\Omega(t)$, productivity is conditionally distributed

$$a(t)|\Omega(t) \sim N\left(a(t-\tau) + \gamma\left(s(t) - s(t-\tau)\right), \nu\right)$$

where

$$\gamma \equiv \frac{\sigma_a^2}{\sigma_a^2 + \sigma_n^2} \qquad \nu \equiv \frac{\tau \sigma_a^2 \sigma_n^2}{\sigma_a^2 + \sigma_n^2}$$

How Do Nowcasts Behave?

Lemma (1.b)

A firm's expected productivity $\hat{a} \equiv \mathbb{E}[a|\Omega]$ and nowcast error u follow the diffusions

$$d\hat{a} = \sigma_a dW^{\hat{a}}$$
 $du = \sigma_u dW^u$

where

$$\begin{split} dW_t^{\hat{a}} &= (1 - \gamma)dW_{t-\tau}^A + \gamma dW_t^A + \gamma \frac{\sigma_n}{\sigma_a} (dW_t^n - dW_{t-\tau}^n) \\ dW_t^u &= (1 - \gamma)\frac{\sigma_a}{\sigma_u} (dW_t^A - dW_{t-\tau}^A) + \gamma \frac{\sigma_n}{\sigma_u} (dW_t^n - dW_{t-\tau}^n) \\ \sigma_u^2 &= 2\frac{\sigma_n^2 \sigma_a^2}{\sigma_a^2 + \sigma_n^2} \end{split}$$

Boundary Solution

The critical value \hat{b} depends on: the variance of nowcast errors ν , the capital share α , the cost of investment ψ , as well as ϱ and m defined as:

$$\varrho \equiv \frac{\mu - \sqrt{\mu^2 + 2\sigma_a^2 r}}{\sigma_a^2} \qquad m \equiv \frac{1}{r + \mu\alpha - \frac{\sigma_a^2}{2}\alpha^2}$$

Lemma (3)

The critical value of expected normalized capital is

$$\hat{b} = \underbrace{\frac{1}{(1-\alpha)} \log \left(\frac{m\alpha(\alpha-\varrho)}{\psi(1-\varrho)} \right)}_{b^{FI} \text{ full info. boundary}} + \underbrace{\frac{\alpha^2 \nu}{2(1-\alpha)}}_{b^{FI} \text{ full info. boundary}}$$

Solving the Firm's Problem: Normalization

Standard approach: define normalized capital

$$X \equiv \frac{K}{A} \qquad \qquad x \equiv k - a$$

• HJB is simpler in one dimension:

$$rV(X) = X^{\alpha} - \delta X V'(X) + \frac{\sigma_a^2 X^2}{2} V''(X)$$

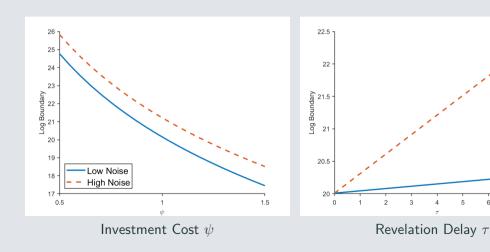
or in logs

$$rv(x) = e^{\alpha x} - \mu v'(x) + \frac{\sigma_a^2}{2}v''(x)$$

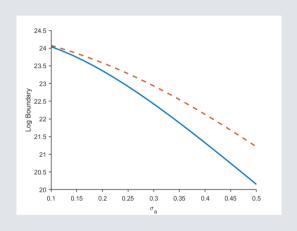
where
$$\mu \equiv \delta + \frac{\sigma_a^2}{2}$$



How the Boundary \hat{b} Depends on the Information Friction

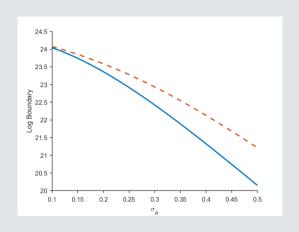


How the Boundary \hat{b} Depends on "Uncertainty"



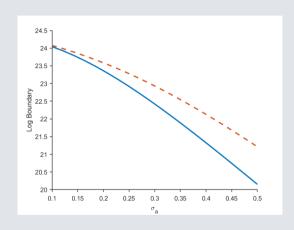
 Full info option-value effect of uncertainty over *future* productivity: higher volatility
 lower capital threshold

How the Boundary \hat{b} Depends on "Uncertainty"



- Full info option-value effect of uncertainty over *future* productivity: higher volatility
 lower capital threshold
- ... but uncertainty over *current* productivity has opposite effect: more noise $(\sigma_n \uparrow) \implies higher$ capital threshold

How the Boundary \hat{b} Depends on "Uncertainty"



- Full info option-value effect of uncertainty over *future* productivity: higher volatility
 lower capital threshold
- ... but uncertainty over *current* productivity has opposite effect: more noise $(\sigma_n \uparrow) \implies higher$ capital threshold
- Noise interacts nonlinearly with the original effect!

• Firm entry/exit keeps the size distribution non-degenerate



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- ullet Firms exit randomly at rate η , with value returned to owners. Measure η of firms enter at every moment



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 - Entering firms are as uncertain about productivity as existing firms: $a \sim N(\hat{a}, \nu)$
 - ullet Their expected normalized capital \hat{x} enters at the critical value \hat{b}



Summary of the Japanese Firm-level Data

Table 1: Sample Comparison (Quarterly)

Moments	Merged Dataset	Entire Sample (FSS)
Number of obs. (Non-missing sales)	392,158	1,260,836
Average employment	1040.582	491.6123
Average sales (million JPY)	19991.75	8541.767
Average fixed capital stock	59919.34	24842.79

Table 2: Investment Moments Using Fixed Capital at Both Frequencies

Frequency	Exit Rate	Agg. Inv. Rate	Inv. Rate Mean	Inv. Rate S.D.	Inaction Rate	Spike Rate
Quarterly	2.00%	1.23%	2.27%	6.10%	60.00%	0.90%
Semiannual	3.96%	2.64%	4.00%	8.3%	36.6%	2.45%



Model Calibration

Table 3: Parametrization of the Stylized Model

Parameter	r	α	τ	ψ	η	ς	δ	σ_{a}	σ_n^0	σ_n^{30}	$\Delta \sigma_n$
Value	1%	0.85	1	1	2%	0	1.23%	0.15	0.00	$0.75\sigma_a$	$0.025\sigma_a$

Table 4: Information Incompleteness and Investment Moments

Industry	σ_n	ξ_s	Inv. Rate Mean	Inv. Rate S.D.	Inaction Rate	Spike Rate
Full Information	0.000	0.000	2.37%	6.7%	81.0%	3.9%
Median Noise	$0.375\sigma_a$	0.018	2.29%	6.1%	79.8%	3.3%
Highest Noise	$0.75\sigma_a$	0.055	2.20%	5.53%	77.7%	2.4%

Partial Irreversibility

• If firms invest, they do so at cost $\Psi(I)$:

$$\Psi(I) = egin{cases} \psi_+ I & I \geq 0 \ \psi_- I & I < 0 \end{cases}$$

with
$$\psi_+ > \psi_- > 0$$

- Instantaneous profit is $\pi = A^{1-\alpha}K^{\alpha} \Psi(I)$
- Optimal firm behavior: for a range of capital values, firms choose to neither invest nor divest. Usual HJB in the inaction region.
- Solving the firm's problem comes down to finding the optimal choice of \hat{B}_L and \hat{B}_U

Partial Irreversibility

Lemma

Under incomplete information, the boundary conditions consist of two value-matching conditions:

$$\hat{V}'(\hat{B}_L) = \psi_+ \qquad \qquad \hat{V}'(\hat{B}_U) = \psi_-$$

and two super contact conditions:

$$\hat{V}''(\hat{B}_L) = 0$$
 $\hat{V}''(\hat{B}_U) = 0$

Partial Irreversibility

Proposition (7)

The critical values of expected normalized capital are

$$\hat{b}_L = b_L^{FI} + rac{lpha^2
u}{2(1-lpha)} \qquad \qquad \hat{b}_H = b_H^{FI} + rac{lpha^2
u}{2(1-lpha)}$$

where b_L^{FI} and b_H^{FI} denote the full information solutions such that $\nu = 0$.

